

# TWO-DIMENSIONAL TRANSMISSION LINE MATRIX (TLM) SIMULATION OF THE ELECTROMAGNETIC FIELDS IN A RECTANGULAR SECTION OF A DISCRETIZED GaAs MESFET CHANNEL WITH ARBITRARY DOPING PROFILE

Salam Dindo and Michel Ney

Department of Electrical Engineering  
University of Ottawa  
Ottawa, Ontario

## ABSTRACT

A two-dimensional lossy shunt TLM network is adapted to simulate the Maxwell field equations of a GaAs MESFET. By discretizing the channel into rectangular sections of single thickness, the new TLM technique is shown, with examples, that it can simulate the calculated electromagnetic fields of an arbitrarily doped channel section.

## INTRODUCTION

The TLM technique cannot be extended into physical modelling of a GaAs MESFET because it cannot directly model either of the transport equations (particle, energy, and momentum conservation equations) which govern the fields in the conductive region, or Poisson's equations which govern the fields in the depletion region [1]. Thus, an indirect modelling approach becomes the only alternative. By segmenting the MESFET channel into rectangular sections of single node with  $\Delta l$ , and length  $b(z, x)$ , as shown in Figure 1, MESFET modelling can then be realized with two procedures:

- (1) TLM modelling of the rectangular sections to enable the electric and magnetic fields to be interrelated by the local conductivity and length through Maxwell equations, and,
- (2) Embedding the mathematical algorithms describing the transport and Poisson's equations to dynamically control the conductivity and length of each of the rectangular sections so that current continuity, depletion zone volume, and applied voltages are enforced.

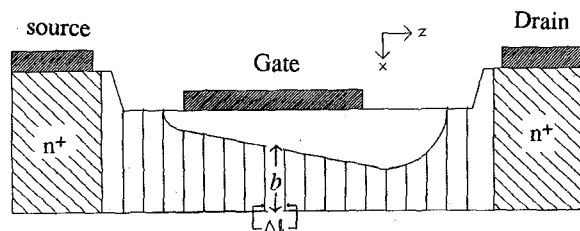


Figure 1: Cross-sectional view of a MESFET structure.

This paper reports on the successful realization of the first procedure.

## THEORY

By adopting the MESFET modelling assumptions:

- (1) the depletion regions is completely empty of carriers,
- (2) the transverse field component  $E_x$  in the channel is zero, and,
- (3) the diffusion of carriers is negligible,

Maxwell's equations in the MESFET are reduced into two dimensions:

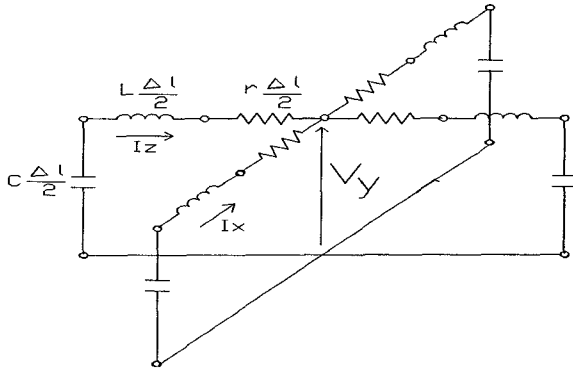
$$\frac{\delta E_z}{\delta x} - \frac{\delta E_x}{\delta z} = \mu \frac{\delta H_y}{\delta t} \quad (1)$$

$$\frac{\delta H_y}{\delta x} = -\epsilon \frac{\delta E_z}{\delta z} - \sigma E_z \quad (2)$$

$$\frac{\delta H_y}{\delta z} = +\epsilon \frac{\delta E_x}{\delta t} + \sigma E_x \quad (3)$$

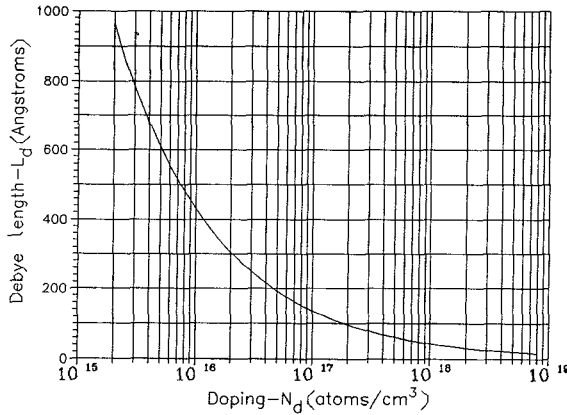
The two-dimensional shunt TLM node in Figure 2 originally used for modelling carrier diffusion[2] is adapted here for the first time to simulate the Maxwell semiconductor equations. With dual modelling in accordance with Babinet's principle, the transverse node currents,  $I_x$  and  $I_z$ , simulate the MESFET's transverse electric fields,  $E_x$  and  $E_z$ , the node voltage  $V_y$ , simulates the longitudinal magnetic field,  $H_y$ , the node resistance,  $r$ , simulates the local conductivity,  $\sigma$ , and the node inductance,  $L$ , simulates the system's dielectric constant,  $\epsilon_r \epsilon_0$ . This TLM node can model conductive media to a large accuracy provided that the loss tangent is kept well below unity. This is achieved by maintaining the Debye length,  $L_d$ , as the upper limit of the node to node spacing  $\Delta$ :

$$\frac{\sigma \Delta l}{\omega \epsilon} \leq \frac{\sigma L_d}{\omega \epsilon} \ll 2 \quad (4)$$

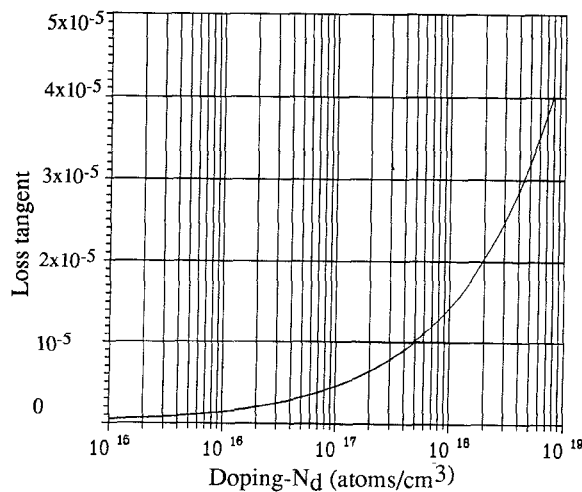


**Figure 2:** TLM node for MESFET analysis

The relation of Debye length to doping in GaAs shown in Figure 3 is used to calculate the loss tangent at 30 GHz. The loss tangent in Figure 4 is well below unity for standard doping levels.



**Figure 3:** Debye length versus GaAs channel doping



**Figure 4:** Loss tangent versus GaAs channel doping

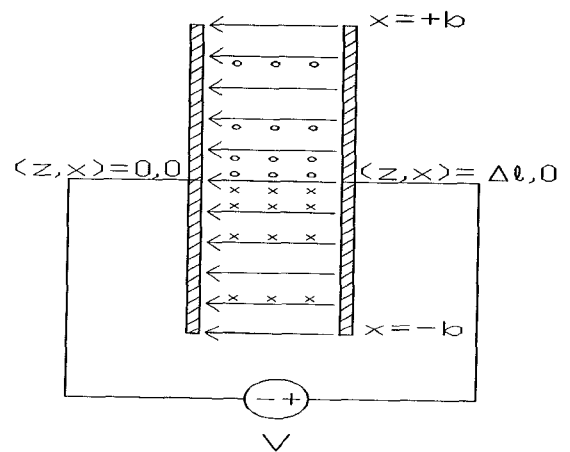
The boundaries of a conductive dielectric rectangular slab must be translated into an electromagnetic equivalent for TLM modelling. One complication arises when attempting to model the base of the channel at the channel/semi-insulating interface where the tangential magnetic and electric fields are non-zero. By converting the structure into a symmetrical one through mirroring the structure at the channel/semi-insulating interface, this complication, as well as the node intensive meshing of the semi-insulating substrate is avoided. Neglecting the semi-insulating substrate in MESFET numerical calculations leads to a small error since the channel/substrate space charge layer contribution to device characteristics is especially little when a buffer layer is used[3]. In a rectangular section of the symmetrical MESFET structure the vertical walls are modelled as perfect electric walls, and the horizontal walls are modelled as perfect magnetic walls.

By considering the rectangular sections as dielectric slabs of arbitrary conductivity distribution,  $\sigma(x)$ , sandwiched by metal contacts and connected to a point DC voltage, the electromagnetic fields can be calculated. The theoretical steady state field distribution for a slab thickness  $\Delta l$  and length  $2b$  is sketched in Figure 5 and is obtained by assuming that the voltage is applied at a single point in the centre of the rectangle. The voltage imposes a steady state electric field:

$$E_z = -\frac{V}{\Delta l} \quad (5)$$

The magnetic field arising from the conduction current density is:

$$\frac{\delta H_y}{\delta x} = -\sigma(x)E_z \quad (6)$$



**Figure 5:** Field distribution in a cross section of a rectangular conductive dielectric. O[x] symbols= magnetic field into [out] of page, arrow symbols= electric field

which upon integration and enforcement of the boundary conditions that the current density on the horizontal boundaries must be zero leads to:

$$H_y = -E_z \int_{x=-b}^{x=0} \sigma(x) dx$$

$$dx = \frac{V}{\Delta l} \int_{x=-b}^{x=0} \alpha(x) dx \text{ for } x < 0 \quad (7)$$

$$H_y = -E_z \int_{x=0}^{x=b} \sigma(x) dx$$

$$dx = \frac{V}{\Delta l} \int_{x=0}^{x=b} \sigma(x) dx \text{ for } x > 0 \quad (8)$$

The transient field responses, on the other hand, are obtained by emulating the MESFET channel as a constant current source. The rectangular section can equivalently be also modelled as a magnetic field source feeding a lossy dielectric represented by a parallel resistor capacitor network. The electric field response to a magnetic field step can be shown to be:

$$E_z(t) = \frac{H_y R}{\Delta l} [1 - e^{-(\frac{t}{\tau})}] \quad (9)$$

The time constant is obtained in terms of the resistance and capacitance per width:

$$\tau = RC = \frac{\Delta l}{2\sigma_{ave}b} \frac{2\epsilon b}{\Delta l} = \frac{\epsilon}{\sigma_{ave}} \quad (10)$$

where  $\sigma_{ave}$  is the average conductivity:

$$\sigma_{ave} = \frac{\int_{x=-b}^{x=b} \sigma(x) dx}{2b} \quad (11)$$

The magnitude of the maximum magnetic field is determined experimentally to depend on the TLM pulse energy, A, and the relative dielectric constant  $\epsilon_r$ :

$$H_y = 2.65 \times 10^{-3} A \sqrt{\epsilon_r} [\text{Amperes/m}] \quad (12)$$

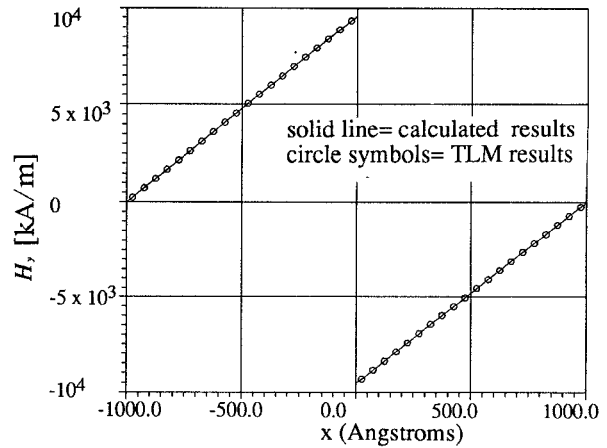
## RESULTS

Comparison of the theoretical and experimental results of TLM method are given in two examples for the cases of (1) uniform and (2) non-uniform channel conductivity. The parameters of the uniformly doped rectangular section of a MESFET channel are shown in Table 1. The theoretical and experimental steady-state spatial magnetic and electric field responses in Figures 6 and 7 show virtually exact agreement. The magnitude of the magnetic field linearly increases to a maximum at the center of the rectangle where the point voltage is applied, while the electric field is spatially constant. The linear distribution of the magnetic field arises from the integration of constant conductivity. The calculated and experimental electric

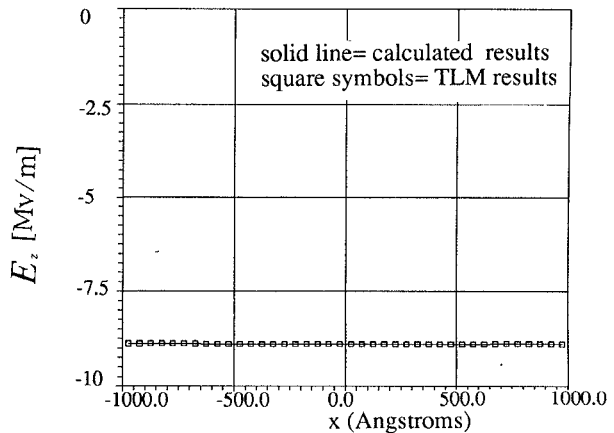
field transient response to the constant magnetic field is shown in Figure 8. The TLM trace exhibits a first order response enveloping decaying second order sinusoids. The origin of the decaying sinusoids can be shown to arise from:

- (1) the impact of impulses into the second order TLM shunt node, and,
- (2) the length of the rectangular sections.

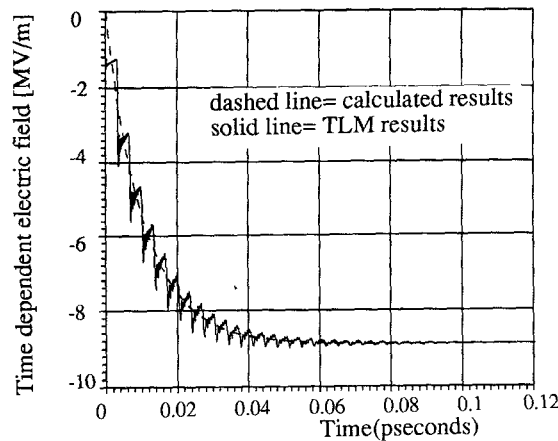
Filtering of the high-frequency components of these ripples is demonstrated to generate the entire calculated response including its time constant. Consider in the second example the case of a linearly doped MESFET channel with the parameters shown in Table 1. The theoretical and experimental steady-state spatial magnetic and electric field responses in Figures 9 and 10 also show virtually exact agreement. In this case, the magnitude of the magnetic field quadratically increases to a maximum at the center of the rectangle while the electric field is still spatially constant. The quadratic distribution of the magnetic field arises from the integration of linearly increasing conductivity. The calculated and experimental electric field transient response to the constant magnetic field is shown in Figure 11.



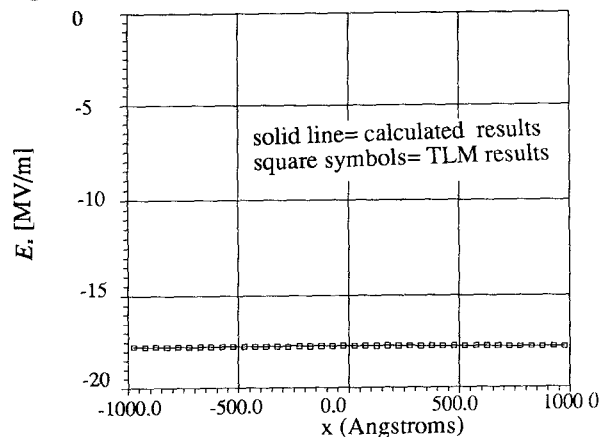
**Figure 6:** Steady-state spatial magnetic field



**Figure 7:** Steady-state spatial electric field



**Figure 8:** Transient response of the electric field



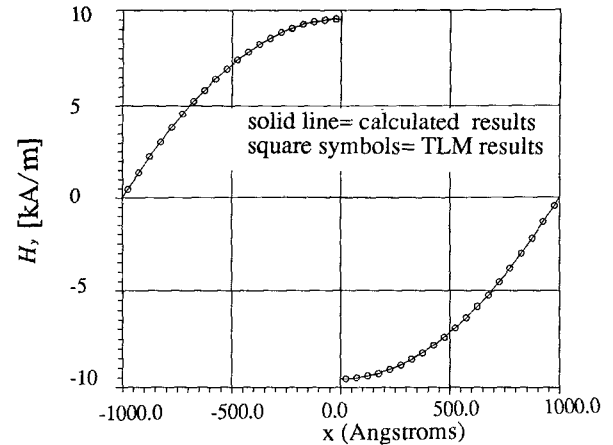
**Figure 10:** Steady-state spatial electric field

	Example 1	Example 2
$b [\text{\AA}]$	1000	1000
$\sigma[\Omega - M]$	10800	$10.8 \times [x \text{ in } \text{\AA}]$
$L_d[\text{\AA}]$	111	111
$\Delta[\text{\AA}]$	50	50
$\Delta t[S]$	$6 \times 10^{-17}$	$6 \times 10^{-17}$
A	$10^6$	$10^6$

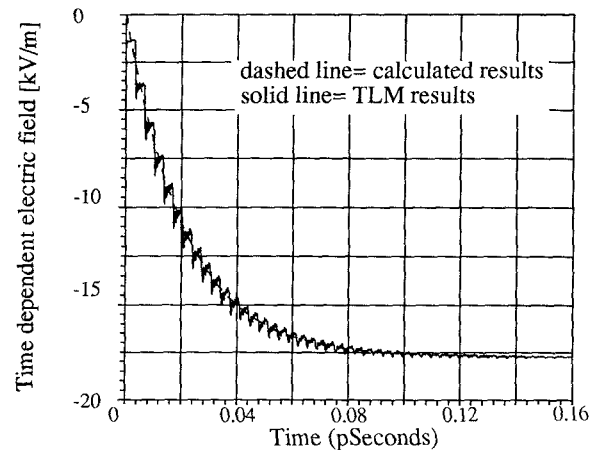
**TABLE 1:** Example 1 & 2 parameters

## CONCLUSION

A TLM technique is successfully applied into the modelling of conductive dielectric rectangular sections of a discretized GaAs MESFET channel. Research will continue to use these results for the full time domain GaAs MESFET modelling by embedding the mathematical algorithms describing the transport and Poisson's equations into the new TLM technique.



**Figure 9:** Steady-state spatial magnetic field



**Figure 11:** Transient response of the electric field

## REFERENCES

- [1] S. Yoganathan, S. Banerjee, T. Itoh, H. Shichijo, and S. El-Ghazaly, "A Numerical Model of GaAs MESFETs Including Energy Balance for Microwave Applications", IEEE Microwave and Guided Wave Letters, Vol. 1 No. 7, pp. 175-177, July 1991.
- [2] SI Pulko, A. Mallik, and P.B. Johns, "Application of TLM to Thermal Diffusion in Bodies of Complex Geometry", International Journal for Numerical Methods in Engineering, Vol. 23, 2303-2312 (1986).
- [3] K. Horio, Y. Fuseya, H. Kusuki, and H. Yanai, "Numerical Simulation of GaAs MESFETs with a P-Buffer Layer on the Semi-Insulating Substrate Compensated by Deep Traps", IEEE Transactions MTT, vol. 37, No. 9 pp. 1371-1379 September 1989.